An LMI-Based Fuzzy State Feedback Control with Multi-Objectives

Sung Kyung Hong*
School of Mechanical & Aerospace Engineering, Sejong University,
98 Kunja-Dong, Kwangjin-Ku, Seoul 143-747, Korea
Yoonsu Nam
Department of Mechanical Engineering, Kangwon National University,
192-1, Hyoja 2 Chunchon, Kangwon-do 200-701, Korea

This paper proposes a systematic design methodology for the Takagi-Sugeno (TS) model based fuzzy state feedback control system with multi-objectives. In this investigation, the objectives are set to be guaranteed stability and pre-specified transient performance, and this scheme is applied to a nonlinear magnetic bearing system. More significantly, in the proposed methodology, the control design problems that consider both stability and desired transient performance are reduced to the standard LMI problems. Therefore, solving these LMI constraints directly (not trial and error) lead to a fuzzy state-feedback controller such that the resulting fuzzy control system meets the above two objectives. Simulation and experimentation results show that the proposed LMI-based design methodology yields not only maximized stability boundary but also the desired transient responses.

Key Words : Fuzzy State Feedback Control, LMI, Multi-Objective

1. Introduction

In the past two decades, Fuzzy Logic Control (FLC) has been proposed as an alternative to the traditional control techniques with many successful applications. In particular, systems which are difficult to model, because of insufficient knowledge of the dynamic characteristics, and nonlinear terms with significant variations in the parameter of the model are attractive candidates for the application of FLC. However it has been argued that FLC being a rule based control strategy, almost by definition, lacks an analytic and systematic methodology for the issues of stability, robustness, and other performance requirements, and therefore, it cannot be reconciled with the traditional methods of control design and analysis.

In recent years, there have been many research efforts on these issues based on the Takagi-Sugeno (TS) model (Takagi, 1985) based fuzzy control (Parallel Distributed Compensator (PDC), following the terminology in (Tanaka, 1994), (Wang, 1996)). The concept of the PDC approach is to design a compensator using linear control design techniques for each TS linear local model. The resulting overall fuzzy controller, which is nonlinear, behaves like a gain-scheduling controller, where the gain-scheduling is implemented with fuzzy logic. For this TS model based fuzzy control system, Wang et al. (Wang, 1996) proved the stability by finding a common symmetric positive definite matrix P for the r subsystems in general and suggested the idea of using Linear Matrix Inequality (LMI) for finding the common P matrix. By introducing the stability issue in fuzzy control, their works have been con-

* Corresponding Author,
E-mail : skhong@sejong.ac.kr
TEL : +82-2-3408-3772; **FAX** : +82-2-3408-3333
School of Mechanical & Aerospace Engineering, Sejong University 98 Kunja-Dong, Kwangjin-Ku, Seoul 143-747, Korea. (Manuscript **Received** August 23, 2002; **Revised** November 4, 2002)

sidered very important results and some refining efforts have been pursued thereafter. However the design process presented in (Tanaka, 1994) and (Wang, 1996) involves an iterative process. That is, for each rule a controller is designed based on consideration of local performance only, then LMI- based stability analysis is carried out to check the global stability condition. In the case that the stability conditions are not satisfied, the controller for each rule should be redesigned. To overcome such a defect, Zhao et al. (Zhao, 1996) pointed out that it is more desirable to directly design a controller (instead of iterative process) which guarantees global stability by recasting to LMI problems. They, however, did not consider performance issues such as transient behaviors. Generally, such a design focused on only stability issue does not directly deal with the desired dynamic characteristic of the closed-loop system, which is commonly expressed in terms of transient responses. In contrast, satisfactory time response and closed-loop damping can be enforced by constraining the closed-loop poles to lie in a suitable subregion of the left-half plane (Chilali, 1996). Motivated by the LMI formulation of pole placement constraint of the conventional state feedback case in (Chilali, 1996), we tried to modify the formulation to apply to the multi-objective TS model based FLC design problem.

In this paper, our main focus is on (1) the extension of the previous LMI-based design methodology for the stable fuzzy control system by imposing the additional requirement of the closed-loop pole locations, and (2) the demonstration of the usefulness of the proposed design methodology via applying it to a regulation problem of a nonlinear magnetic bearing system. Especially, among many serious problems pertaining to magnetic bearings, the gap nonlinearity of magnetic force is dealt with. The same model that has been studied previously in (Hong, 1999), (Hong, 1997) and (Hong, 2000) is used.

This paper is organized into five sections. The next section introduces the background materials concerning TS fuzzy model and model-based fuzzy controller. Section III describes the formulations of the LMI-based fuzzy state feedback controller for the stability and the closed-loop pole location requirements. In Section IV, simulation studies and experimental results are presented by the application of the proposed methodology to the nonlinear magnetic bearing system. Concluding remarks are given in Section V.

2. TS Fuzzy Model and Control

2.1 TS fuzzy model

An nth order SISO nonlinear system can be expressed in the following form :

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x_1, x_2, \cdots, x_n, u) \end{aligned} \tag{1}$$

where, u is the control input. By taking the Taylor's series expansion of Eq. (1) for r operating points (x_l^*, u^*), where $l=1, 2, \cdots, r$, the nonlinear system can be represented by the following linearized state space form with the bias term di induced from the model linearization :

$$\dot{x} = A_i x + B_i u + d_i, \; i=1, 2, \cdots, r \tag{2}$$

where,

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 1 \\ \dfrac{\partial f(x_l^*, u^*)}{\partial x_1} & \dfrac{\partial f(x_l^*, u^*)}{\partial x_2} & \cdots & \dfrac{\partial f(x_l^*, u^*)}{\partial x_n} \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \dfrac{\partial f(x_l^*, u^*)}{\partial u} \end{bmatrix}$$

$$d_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ f(x_l^*, u^*) - \sum_{l=1}^{n} \dfrac{\partial f(x_l^*, u^*)}{\partial x_l} x_l^* - \dfrac{\partial f(x_l^*, u^*)}{\partial u} u^* \end{bmatrix}$$

and the variables with $*$ denote the values at the operating points.

The continuous fuzzy dynamic model is described by fuzzy If-Then rules to express local linear input-output relations of nonlinear systems around each operating point by the above linear local model. The ith rule of this fuzzy model is of the following form:

$$\begin{aligned}&\text{If } x_1(t) \text{ is } L_{i1} \text{ and } \cdots x_n(t) \text{ is } L_{in} \text{ and } u(t) \text{ is } M_i,\\&\text{Then } \dot{x}(x)=A_i x(t)+B_i u(t)+d_i\end{aligned} \tag{3}$$

$i=1, 2, \cdots, r$ and r is the number of rules and L_{ij} and M_i are fuzzy sets centered at the ith operating point. The categories of the fuzzy sets are expressed as NE, ZE, and PO, where NE represents negative, ZE zero, and PO positive. The inference performed via the TS model is an interpolation of all the relevant linear models. The degree of relevance becomes the weight in the interpolation process. It should be noted that even if the rules in a TS fuzzy model involve only linear combinations of the model inputs, the entire model is truly nonlinear as shown in (4) below.

Given a pair of (x_i, u), the final output of the fuzzy system is given by the equation below:

$$\dot{\mathbf{x}}=\frac{\sum_{i=1}^{r} W_i\{A_i x+B_i u+d_i\}}{\sum_{i=1}^{r} W_i} \tag{4}$$

where, $W_i=\prod_{j=1}^{n}(L_{ij}(x_j)\cdot M_i(u))$, $L_{ij}(x_j)$, and $M_i(u)$ are the grades of membership of x_j and u in L_{ij} and M_i, respectively.

2.2 TS model-based fuzzy control

The concept of PDC, following the terminology (Tanaka, 1994), (Wang, 1996), is utilized to design fuzzy state-feedback controllers on the basis of the TS fuzzy models (3). Linear control theory can be used to design the consequent parts of the fuzzy control rules, because the consequent parts of TS fuzzy models are described by linear state equations. If we compute the control input u to be

$$u=\tilde{u}-K_{0i} \tag{5}$$

where, $K_{0i}=d_i(1, n)/B_i(1, n)$, then the Eq. (2) is described by

$$\dot{x}=A_i x+B_i\tilde{u},\ i=1, 2, \cdots, r. \tag{6}$$

Based on the revised piecewise linear model (6), we determine a state feedback controller described by

$$\tilde{u}=K_i x \tag{7}$$

where, K_i is a feedback gain matrix to be chosen for the ith operating point via proper design methodologies. It should be noted that, however, the value of the control input actually used in the fuzzy rules would be derived from Eq. (5). Hence a set of r control rules takes the following form:

$$\begin{aligned}&\text{If } x_1(t) \text{ is } L_{x1l} \text{ and } \cdots x_n(t) \text{ is } L_{xni} \text{ and } u(t) \text{ is } L_{ui},\\&\text{Then } u(t+1)=K_i x(t)\end{aligned} \tag{8}$$

where, the index $t+1$ in the consequent part is introduced to distinguish the previous control action in the antecedent part in order to avoid algebraic loops. Each of the rules can be viewed as describing a "local" state-feedback controller associated with the corresponding "local" submodel of the system to be controlled. The resulting total control action is

$$u=\frac{\sum_{i=1}^{r} W_i\cdot(K_i x-K_{0i})}{\sum_{i=1}^{r} W_i}. \tag{9}$$

Note that the resulting fuzzy controller (9) is nonlinear in general since the coefficient of the controller depends nonlinearly on the system input and output via the fuzzy weights. Substituting (9) into (4), the fuzzy control system (closed-loop), shortly FCS, can be represented by

$$\dot{x}=\frac{\sum_{i=1}^{r}\sum_{j=1}^{r} W_i W_j\{A_i+B_i K_j\}}{\sum_{i=1}^{n}\sum_{j=1}^{r} W_i W_j}x. \tag{10}$$

3. An LMI-Based Fuzzy State Feedback Control System Design

3.1 LMI formulation for stability requirement

A sufficient quadratic stability condition derived by Tanaka and Sugeno (Tanaka, 1992) for ensuring stability of (10) is given as follows:

Theorem 1. The fuzzy control system (10) is quadratically stable for some stable feedback K_j (via PDC scheme) if there exists a common positive definite matrix P such that

$$\begin{aligned}&\{A_i+B_i K_j\}^T P+P\{A_i+B_i K_j\}<0\\&i, j=1, 2, \cdots, r.\end{aligned} \tag{11}$$

Note that system (10) can be also rewritten as

$$\dot{x}=\frac{\sum_{i=1}^{r}\sum_{j=1}^{r} W_i W_j G_{ii}x+\sum_{i<j}^{r} W_i W_j G_{ij}x}{\sum_{i=1}^{r}\sum_{j=1}^{r} W_i W_j} \tag{12}$$

where, for $G_{ii}=A_i+B_iK_i$ for $i=j=1, 2, \cdots, r$, and $G_{ij}=\frac{(A_i+B_iK_j)+(A_j+B_jK_i)}{2}$ for $i<j$.

By applying Theorem 1, we have the following revised sufficient condition for the fuzzy control system (12).

Theorem 2. The fuzzy control system (10) is quadratically stable for some state feedback K_j (via PDC scheme) if there exists a common positive definite matrix P such that

$$\begin{aligned} &G_{ii}^T P+PG_{ii}<0 \quad i=1, 2, \cdots, r, \\ &G_{ij}^T P+PG_{ij}<0 \quad i<j\le r. \end{aligned} \tag{13}$$

Conditions (13) are not jointly convex in K_j's and P. To cast these conditions into LMIs, we define $Q=P^{-1}$. Then we can rewrite (13) as:

$$\begin{aligned} &QG_{ii}^T+G_{ii}Q<0 \quad i=1, 2, \cdots, r, \\ &QG_{ij}^T+G_{ij}Q<0 \quad i<j\le r. \end{aligned} \tag{14}$$

Our objective is to design the gain matrix K_j ($i=1, 2, \cdots, r$) such that conditions (14) can be satisfied. This is the 'quadratic stabilizability' problem. If such a gain K_j exists, the system is said to be quadratic ally stabilizable.

3.2 LMI formulation for pole-placement requirement

In the synthesis of a control system, meeting some desired performances should be considered along with stability. Generally, stability condition (Theorem 2) does not directly deal with the transient responses of the closed-loop system. In contrast, a satisfactory transient response of a system can be guaranteed by confining its poles in a prescribed region. This section discusses a Lyapunov characterization of pole clustering regions in terms of LMIs. To this purpose, we introduce the following LMI-based representation of stability regions.

Definition 1. LMI Stability Region (Chilali, 1996). A subset of D of the complex plane is called an LMI region if there exits a symmetric

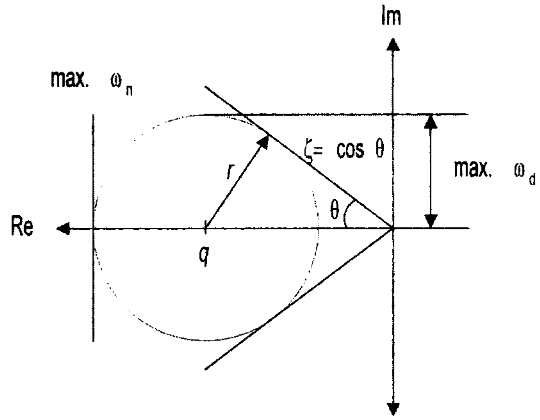

Fig. 1 Circular region (D) for pole locations

matrix $\alpha=[\alpha_{kl}]\in R^{m\times m}$ and a matrix $\beta=[\beta_{kl}]\in R^{m\times m}$ such that

$$D=\{z\in C : f_D(z)<0\} \tag{15}$$

where the characteristic function $f_D(z)$ is given by $f_D(z)=[\alpha_{kl}+\beta_{kl}z+\beta_{kl}\bar{z}]_{1\le k,l\le m}$ (f_D is valued in the space of $m\times m$ Hermitian matrices). It is easily seen that LMI regions are convex and symmetric with respect to the real axis. Specifically, we consider a circle LMI region D

$$D=\{x+jy\in C : (x+q)^2+y^2<r^2\} \tag{16}$$

centered at $(-q, 0)$ and has radius $r>0$, where the characteristic function is given by

$$f_D(z)=\begin{pmatrix} -r & \bar{z}+q \\ z+q & -r \end{pmatrix}. \tag{17}$$

As shown in Fig. 1, if $\lambda=-\varsigma\omega_n\pm j\omega_d$ is a complex pole lying in D with damping ratio ζ, undamped natural frequency ω_n, damped natural frequency ω_d, then $\varsigma>\sqrt{1-(r^2/q^2)}$, $\omega_n<q+r$, and $\omega_d<r$. This circle region puts a lower bound on both exponential decay rate and the damping ratio of the closed-loop response, and thus is very common in practical control design. Motivated by Chilali and Cabinet's Theorem (Chilali, 1996), an extended Lyapunov Theorem for the fuzzy control system (10) is developed with the above definition of an LMI-based circular pole region as below.

Theorem 3. The fuzzy control system (10) is D-stable (all the complex poles lying in LMI region D) if and only if there exists a positive symmetric matrix Q such that

$$\begin{pmatrix} -rQ & qQ+Q\{A_i+B_iK_j\}^T \\ qQ+\{A_i+B_iK_j\}Q & -rQ \end{pmatrix}<0 \quad (18)$$

The proof and more details of this theorem can be found in (Chilali, 1996).

It should be noted that since Theorem 3 will be used for the supplementary constraints in our problem, constraints of the LMI region to both cases of $i=j$ and $i<j$ may not be necessary: it suffices to locate the poles of only dominant term (in the case of $i=j$) in the prescribed LMI regions.

2.3 Formulation for the synthesis

In this section, we formulate a problem for the design of a fuzzy state feedback control system that guarantees stability and satisfies desired transient responses by using the above LMI constraint (14) and (18). With change of variable $Y_i=K_iQ$ and substituting into (14) and (18), there lead to the following LMI formulation of our fuzzy state-feedback synthesis problem.

Theorem 4. The fuzzy control system (10) is stabilizable in the specified region D via PDC scheme if there exists a common $Q>0$ and Y_i such that the following LMI conditions hold:

$$\begin{gathered} A_iQ+QA_i^T+B_iY_i+Y_i^TB_i^T<0 \\ \frac{A_iQ+QA_j^T+B_iY_j+Y_j^TB_i^T}{2}+\frac{A_jQ+QA_i^T+B_jY_i+Y_i^TB_j^T}{2}<0 \\ \begin{pmatrix} -rQ & qQ+QA_i^T+Y_i^TB_i \\ qQ+A_iQ+B_iY_i^T & -rQ \end{pmatrix}<0 \end{gathered} \quad (19)$$

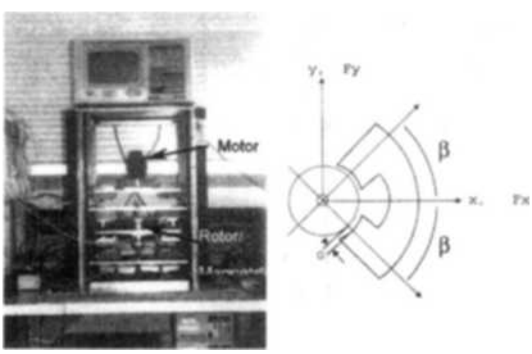

Fig. 2 Laboratory magnetic bearing experimental setup

Given a solution (Q, Y_i), the fuzzy state feedback gain is obtained by

$$K_i=Y_iQ^{-1} \quad (20)$$

As a result, the obtained gain guarantees global stability while it provides desired transient behaviors by constraining the closed-loop poles of the locally linearized systems in the region D. Admittedly with some degree of conservatism, these results offer numerically tractable means of performing multi-objective fuzzy state-feedback controller design.

4. Application to an Active Magnetic Bearing System

The objective of the control system of an active magnetic bearing (AMB) is to maximize the stable boundary of operation with desired transient performance through overcoming the gap nonlinearity. To achieve such an objective, we design a fuzzy state-feedback controller based on Theorem 4 for a nonlinear AMB system. The validity and practicality of the obtained controller is demonstrated through simulations and experiments. The model that has been studied previously in (Hong, 1997) and (Hong, 2000) is used.

4.1 Active magnetic bearing (AMB) system

The AMB system employed in this research is a two-axis controlled vertical shaft magnetic bearing with a symmetric structure. An outline of this system is depicted in Fig. 2. Due to the small gyroscopic effect of this setup (Hong, 2000), the system can be divided into two identical subsystems (x-z and y-z planes), which means that each gap displacement for the x-direction and y-direction can be controlled individually. Thus, without loss of generality, we will focus our analysis strictly on the x-direction motion only.

The equations of motion for the AMB can be represented as (Hong, 2000):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \left(\frac{l^2k}{J_T}\right)\left(\frac{(i_b+i_p)^2}{(G-\beta x_1)^2}-\frac{(i_b-i_p)^2}{(G+\beta x_1)^2}\right) \end{aligned} \quad (21)$$

where, x_1 denotes the displacement of the rotor from the center position, x_2 is the velocity, and i_p is the control input current applied to the electromagnets. The physical parameters of this experimental setup are given as follows:

k (force constant): 0.00186 $lb \cdot in^2/A^2$
β (sensitivity of air gap to shaft displacement.): 0.974
i_b (bias current): 0.3 A
G (nominal air gap): 0.02 in
l (length of the rotor): 4.8 in
J_T (transverse MOI of the rotor): 0.134 $lb \cdot in \cdot s$

4.2 TS fuzzy model for the AMB

We represent the nonlinear system (21) by a TS fuzzy model (3) via linearization (using Taylor's series expansion) around several operating points (Hong, 1999), (Hong, 2000). With considerations of the nonlinear dynamic characteristics of the AMB (Hong, 2000) shown in Fig. 3, the membership functions of the fuzzy sets for x_l, and u $(=i_p)$ are defined as Fig. 4.

With this definition we have totally $3^2=9$ rules.

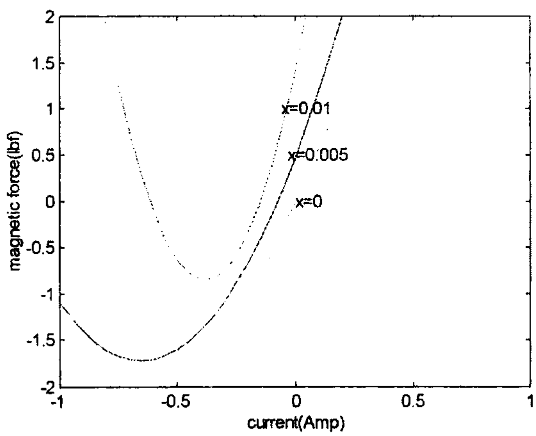

Fig. 3 2-D View of the magnetic force characteristic

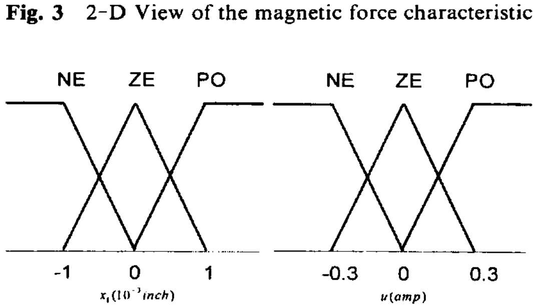

Fig. 4 Membership function

However, to reduce the number of fuzzy rules, the rules with similar antecedents and same consequent were grouped together and described by a single approximate rule. As a result, three rules are used to describe nonlinear dynamics (21). Denoting $x=[x_1\ x_2]'$, the piecewise linear TS fuzzy model can be written as:

Plant Rule 1: If $x_1(t)$ is ZE,
Then $\dot{x}(t)=A_1x+B_1u+d_1$
Plant Rule 2: If $x_1(t)$ is PO(or NE)
and $u(t)$ is ZE,
Then $\dot{x}(t)=A_2x+B_2u\pm d_2$
Plant Rule 3: If $x_1(t)$ is PO(or NE)
and $u(t)$ is NE(or PO),
Then $\dot{x}(t)=A_3x+B_3u\pm d_3$

where,

$$A_1=\begin{bmatrix} 0 & 1 \\ 16506 & 0 \end{bmatrix},\ A_2=\begin{bmatrix} 0 & 1 \\ 63640 & 0 \end{bmatrix},\ A_3=\begin{bmatrix} 0 & 1 \\ 63640 & 0 \end{bmatrix}$$

$$B_1=\begin{bmatrix} 0 \\ 1130 \end{bmatrix},\ B_2=\begin{bmatrix} 0 \\ 2402 \end{bmatrix},\ B_3=\begin{bmatrix} 0 \\ 511 \end{bmatrix}$$

$$d_1=\begin{bmatrix} 0 \\ 0 \end{bmatrix},\ d_2=\begin{bmatrix} 0 \\ 352.7 \end{bmatrix},\ d_3=\begin{bmatrix} 0 \\ 100.4 \end{bmatrix}.$$

4.3 Synthesis of fuzzy control system

Using Theorem 4, we can design a fuzzy state feedback controller that guarantees global stability while provides desired transient behavior by locating the closed-loop poles in D. The stability region D is a circle of center $(-q, 0)$ and radius r and the LMI synthesis is performed for a set of values:

$$(q,\ r)=(450,\ 250)$$

which constraint the transient response by the damping ratio as $\zeta>0.83$, and rise time as $0.0025<t_r<0.009$. Then the LMI region has the following characteristic function as

$$f_D(z)=\begin{pmatrix} -250 & 450+z \\ 450+z & -250 \end{pmatrix}.$$

By solving the LMI feasibility problem of Theorem 4, we can obtain a positive symmetric matrix Q as

$$Q=\begin{pmatrix} 0.0001 & -0.0158 \\ -0.0158 & 4.8053 \end{pmatrix}.$$

And Y_1, Y_2 and Y_3 as

$Y_1=[0.0021\ \ -1.0435]$, $Y_2=[0.0002\ \ -0.2052]$, $Y_3=[0.0049\ \ -2.2716]$.

Finally, the state feedback gain can be obtained by (20)

$K_1=[-108.49\ \ -0.57]$, $K_2=[-71.81\ \ -0.2052]$, $K_3=[-212.34\ \ -1.17]$.

For the comparison, we also calculate the state feedback gains when the constraint for the pole-placement is omitted (i.e., considering only stability condition). At that time a positive symmetric matrix Q, matrix Y_i, and gain matrix K_i are as belows:

$$Q=\begin{pmatrix} 0.0007 & -0.0414 \\ -0.0414 & 7.3873 \end{pmatrix}$$

$Y_1=[-0.0161\ \ 0.5540]$, $Y_2=[-0.0203\ \ 1.0659]$, $Y_3=[-0.0229\ \ 0.6460]$

$K_1=[-31.00\ \ -0.10]$, $K_2=[-34.35\ \ -0.05]$, $K_3=[-46.12\ \ -0.17]$

The resulting fuzzy control law for each piecewise linear segment of the fuzzy model can be written as follows:

Control Rule 1 : If $x_1(t)$ is ZE,
Then $u(t+1)=K_2x-K_{01}$

Control Rule 2 : If $x_1(t)$ is PO(or NE)
and $u(t)$ is ZE,
Then $u(t+1)=K_2x-K_{02}$

Control Rule 3 : If $x1(t)$ is PO(or NE)
and $u(t)$ is NE(or PO),
Then $u(t+1)=K_3x-K_{03}$

where, $K_{01}=0$, $K_{02}=0.147$, and $K_{03}=0.197$.

The resulting total control action can be written as (for the positive rotor displacement):

$$u(t+1)=\left(\frac{W_1K_1+W_2K_2+W_3K_3}{W_1+W_2+W_3}\right)x + \left(\frac{W_1K_{01}+W_2K_{02}+W_3K_{03}}{W_1+W_2+W_3}\right).$$

5. Simulations and Experimental Results

5.1 Simulations

To investigate the effectiveness of the proposed controller, some simulations were performed. For the perpose of comparison, another fuzzy state feedback controller that was obtained by stability constraints only (without pole-placement constraints) was employed. It can be noticed that the results of the fuzzy controller obtained by both stability and pole placement constraints (Fig. 5) indicate better transient performance than those of another fuzzy controller obtained by stability constraints only (Fig. 6), while both fuzzy controllers give stable response regardless of any initial displacement. Therefore, it is desirable to tune the stability and transient response simultaneously by combining these two objectives.

The performances of these two controllers were measured by the following quadratic error index

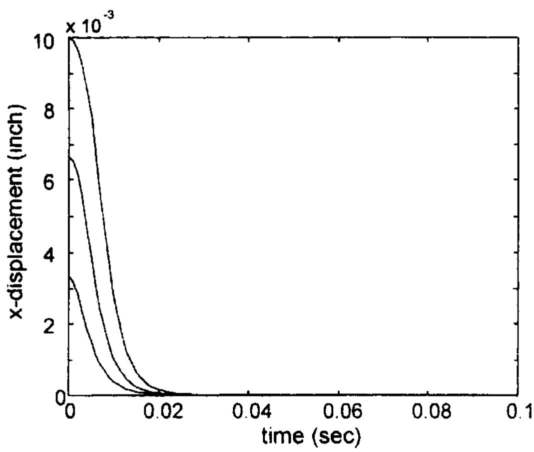

Fig. 5 Fuzzy control with both stability and pole placement constraints

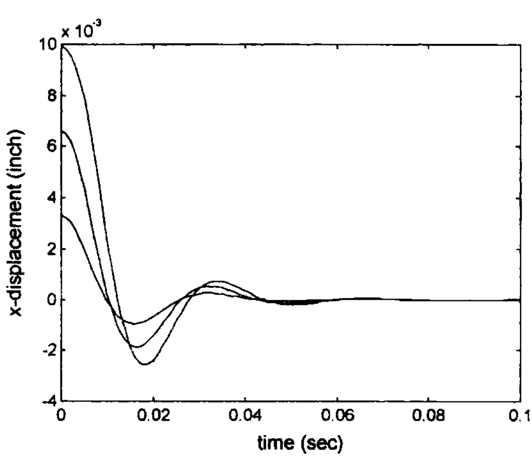

Fig. 6 Fuzzy control with stability constraints only

Table 1 Comparison of two LMI approaches

Design Constraints	Quadratic Error (inch)		
	$x(0)=$ 0.0033	$x(0)=$ 0.0066	$x(0)=$ 0.0099
• Stability Only	5.934e-05	2.541e-04	6.841e-04
• Stability+ Pole-Placement	4.292e-05	2.120e-04	6.338e-04

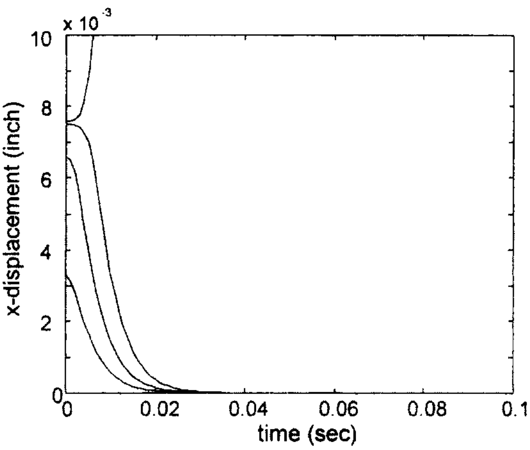

Fig. 7 Conventional linear local control

$$I_{qe}=\int_0^t e(\tau)^2 d\tau. \tag{22}$$

The above performance index was calculated for each response of three different initial displacements. They are summarized in Table 1. Through these results we can verify the effectiveness of the proposed multi-objective (stability & closed-loop pole location) design approach.

We also tested the performance of the linear local controller (control rule 1) which was designed for a single equilibrium point. As can be seen in Fig. 7, it performs well near the equilibrium point, but its effectiveness deteriorates outside of the limited operating region and fails to regulate the rotor for the initial displacement of $x(0)=0.0075$. This small boundary of stability is due largely to the nonlinearity of the AMB.

5.2 Experimentation

In our experiments, position feedback is obtained from position probes located in the stator. The velocity is obtained by differentiating the position signal. A third-order Butterworth filter

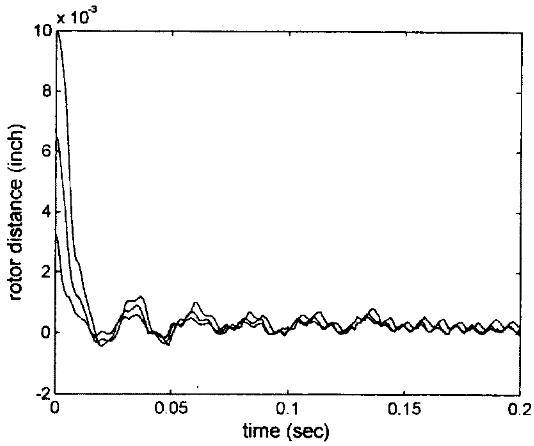

Fig. 8 Experiments of fuzzy control with both stability and pole placement constraints

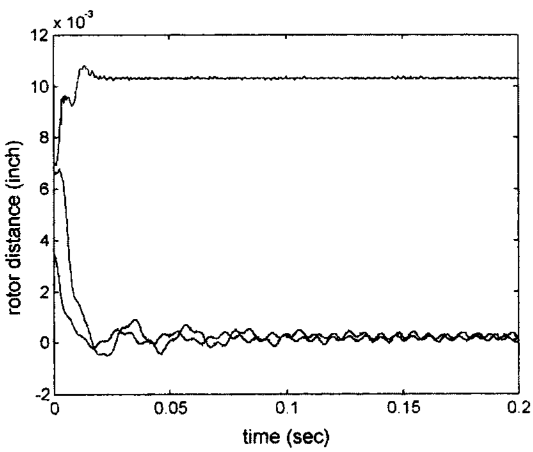

Fig. 9 Experiments of conventional linear local control

with corner frequency of 200 Hz was used to reduce noise in the resulting velocity signal. The sampling frequency is 6000 Hz. Figure 8 shows that the fuzzy controller obtained stable responses with acceptable transient performance regardless of any initial position. On the other hand, as shown in Fig. 9, the linear local controller fails to regulate the rotor around initial displacement of $x(0)=0.007$ in. These results are coinciding with the simulation results. In both Fig. 8 and Fig. 9, it can be seen that some oscillations appear at the transient region and gradually die out as it reaches steady state. The sources of such oscillations are possibly due to the flexibility of the thin rod which is attached between motor and rotor. Therefore extensions of the proposed control scheme to a flexible dynamic system to achieve more sophisticated performance may be interesting possibilities.

6. Conclusion

In this paper, a systematic design methodology for the fuzzy control of nonlinear AMB system with guaranteed stability and pre-specified transient performance is presented. The framework is based on a Takagi-Sugeno fuzzy model and a parallel distributed compensation (PDC) scheme. More significantly, in the proposed methodology, the control design problems which considers both stability and desired transient performance are reduced to the standard LMI problems. Therefore solving these LMI constraints directly (not trial and error) leads to a fuzzy state-feedback controller such that the resulting fuzzy control system meets above two objectives. As a result, this approach is superior to other existing approaches, which achieves the desired control performances by trial and error. Simulation and experimentation results showed that the multi-objective nonlinear fuzzy controller proposed in this paper yields not only maximized stability boundary but also better transient performance than those of another fuzzy controller which was obtained by stability constraints only. As further studies, the extensions of the proposed control scheme to the problems common to all mechanical systems with rotating rotor such as cross-coupling caused by gyroscopic effect and vibration caused by external disturbances will be the possibilities.

Acknowledgment

This work was Supported by Korea Research Foundation Grant. (KRF 2000-005-E00001)

References

Chilali, M. and Gahinet, P., 1996, "H_∞ Design with Pole Placement Constraints: An LMI Approach," *IEEE Trans. Automatic Control*, Vol. 41, No. 3, pp. 358~367.

Gahinet, P, Nemirovski, A., Laud, A. and Chilali, M, 1995, *The LMI Control Toolbox*, The Math Works, Inc., 1995.

Hong, S. K. and Langari, R., 1997, "Experiments on Fuzzy Logic Based Control of a Magnetic Bearing System," in *Proc. North American Fuzzy Information Processing Society* (Syracuse, New York), pp. 90-95.

Hong, S. K. and Langari, R., 1999, "Fuzzy Modeling and Control of a Nonlinear Magnetic Bearing System," *Journal of Intelligent & Fuzzy Systems*, Vol. 7, No. 1, pp. 335~346.

Hong, S. K. and Langari, R., 2000, "Robust Fuzzy Control of a Magnetic Bearing System Subject to Harmonic Disturbances," *IEEE Trans. on Control System Technology*, Vol. 8, No. 2, pp. 366~371.

Hong, S. K. and Langari, R., 2000, "An LMI-Based H-infinitive Fuzzy Control System Design with TS Framework," *Information Sciences : An International Journal*, Vol. 123, No. 1, pp. 163~179.

Takagi, T. and Sugeno, M., 1985, "Fuzzy Identification of Systems and its Application to Modeling and Control," *IEEE Trans. Syst., Man, Cybern.*, Vol. 15, pp. 116~132.

Tanaka, K. and Sugeno, M., 1992, "Stability Analysis and Design of Fuzzy Control systems," *Fuzzy Sets Syst.*, Vol. 45, No. 2, pp. 135~156.

Tanaka, K. and Sano, M., 1994, "A Robust Stabilization Problem of Fuzzy Control Systems and its Application to Backing up Control of a Truck-Trailer," *IEEE Trans. on Fuzzy Systems*, Vol. 2, No. 2, pp. 119~134.

Wang, H. O., Tanaka K. and Griffin, M. F., 1996, "An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues," *IEEE Trans. on Fuzzy Systems*, Vol. 4, No. 1, pp. 14~23.

Zhao, J., Gorez, R. and Wertz, V., 1996, "Synthesis of Fuzzy Control Systems with Desired Performances," *in Proc. IEEE Int. Symp. Intelligent Control*, pp. 115~120, Dearborn, MI